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THEORETICAL ANALYSIS OF SEMI CIRCULAR CURVED BEAM SUBJECTED TO OUT-OF-PLANE LOAD

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ABSTRACT

Curved beams are used as machine or structural members in many applications. They can be classified into two categories based on application of load. Curved beams subjected to In-Plane loads are more familiar and are used for crane hooks, C-clamps etc. The other categories of curved beams are the ones that are subjected to out-of-plane loads. They find applications in automobile universal joints, raider arms and many civil structures etc. The formulations with respect to curved beams of second category are not found in literature. The results of this research on semicircular curved beam subjected to out-of-plane loads have revealed some interesting results. For semicircular curved beams subjected to out-of-plane loads, it is shown that every section is subjected to a combination of transverse shear force, bending moment and twisting moment. Maximum principal stress occurs at a section 120 degrees from the section containing the loading line. Moreover it is observed that fixed end of this curved beam is subjected to a state of pure shear.

KEYWORDS: Combined Torsion and Bending Stress, Curved Beams, Out-of-Plane Load, Stress Distribution

INTRODUCTION

Curved beams are the parts of machine members found in C clamps, crane hooks, frames of presses, punching machines, planers automobile components etc. In straight beams the neutral axis of the section coincides with its centroidal axis and the stress distribution in the beam is linear. But in the case of curved beams the neutral axis of is shifted towards the centre of curvature of the beam causing a non linear distribution of stress. Fonseca et al. [1] studied curved pipes subjected to in-plane loads, Stefano Lenci et al.[2] a 3-d mechanical model of curved beam is analysed by them, Saffari et al.[4] sudies by using circular arc element based on trignometric functions foe in-plane loads, Clive et al.[5] investigated end loaded shallow curved beams of in-plane load type, Öz et al.[6] analysed in plane vibrations of curved beam having open crack, Aimin Yu et al.[7] made a work on naturally twisted curved beams of thin walled sections that of in plane loads.

From the formulations available for curved beams subjected to in-plane loads it is clear that at any section there will be a direct normal or shear stress along with bending stress. We have the formulations to determine stresses for this case.

Stress analysis of curved beams subjected to out-of-plane loads also is important as such beams are used in many machine and structural applications. An explicit formulation for curved beam subjected to out-of-plane load is not found in literature. This paper attempts to determine the stresses induced in such a curved beam.

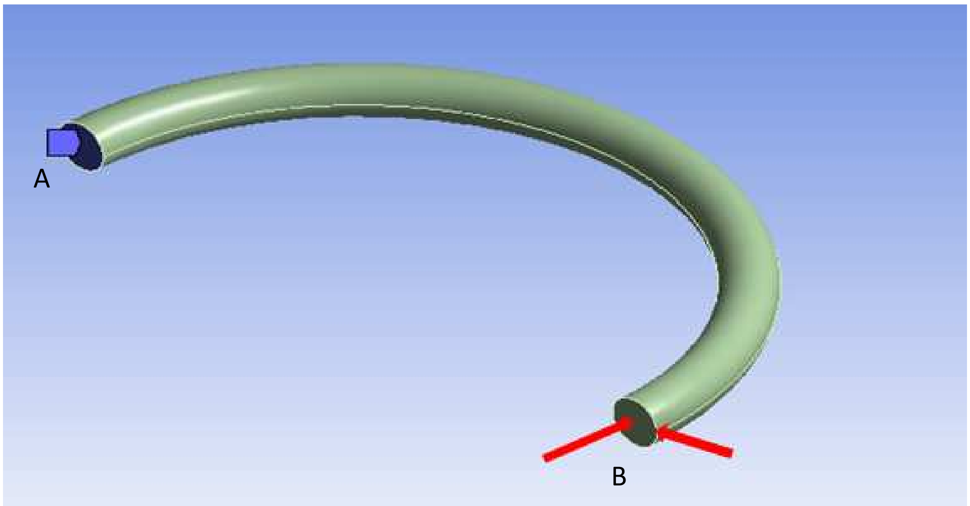

Figure 1: Curved Beam with In-Plane Load

Some of the assumptions made to derive expression of principle stress for curved beam subjected to Out-Of-Plane load case are as follows,

- The radius of curvature is assumed much larger than the section radius.
- The material is assumed to be linearly elastic.
- The beam is assumed to be geometrically planar, i.e., the un-deformed axis of the beam is assumed to be a circle lying in the plane of the beam.
- The cross section is assumed to be constant and with the same orientation with respect to the plane of the beam, so that there is no initial torsion.

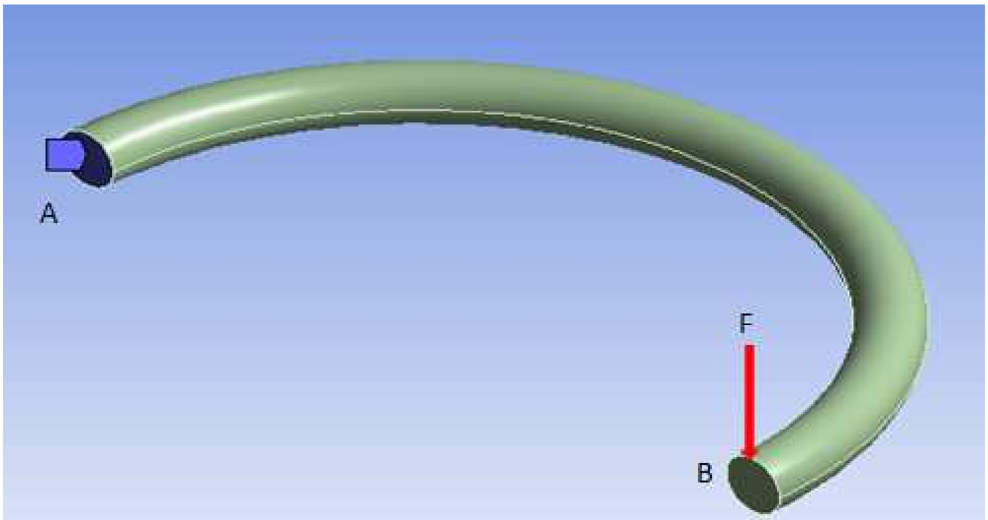

Figure 2: Curved Beam with Out-of-Plane Load

FORMULATION FOR OUT-OF-PLANE LOAD CONDITION

Consider a semi circular curved beam of circular cross section lying in the plane of paper as shown in figure 3(a). The beam is fixed at one end 'A' and an out-of-plane load 'F' is applied at the other end 'B'.

F = Applied load in N

R_o = Outer radius of beam in mm.

R_m = Mean radius of beam in mm.

R_i =Inner radius of beam in mm.

α = angle made by the section X-X w.r.t loading line.

d = diameter at any section X-X .

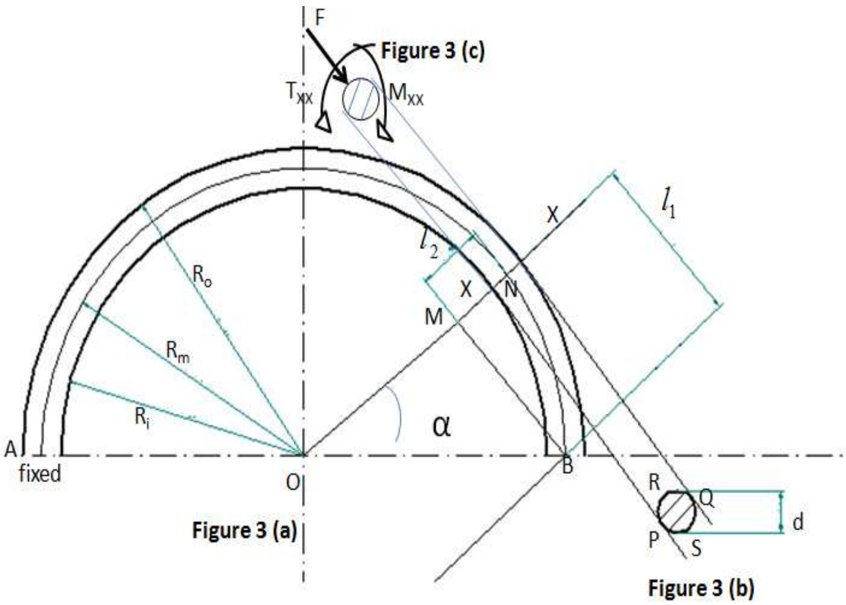

Figure 3(a): Detailed View of Semi Circular Beam
Figure 3(b): Cross Section of Beam at X-X with Extreme Points Indicated
Figure 3(c): Loads and Moments Acting on Cross Section X-X

Let X-X be a plane passing through the centre of curvature and perpendicular to cross section of the beam. Let the angle made by this plane X-X wit respect to the free end be α as shown in fig 3(b). The effect of Out-Of-Plane load F at the section is to cause i). Transverse shear due to direct force F. ii) A bending moment M_{XX} and iii) A twisting moment T_{XX} as shown in fig 3(c). The magnitudes of the stresses due to these loads can be determined as follows:

Let N be the point of intersection of plane X-X and centroidal axis of curve beam. From the point B at the free end where the Out-Of-Plane load is acting, draw a line perpendicular to the line ON as shown in figure 3. Let BM= l_1 and MN= l_2 . Then the bending moment M_{XX} at section X-X is given by

$$M_{XX} = F \times l_1 \tag{1}$$

And twisting moment T_{XX} at section X-X is

$$T_{XX} = F \times l_2 \tag{2}$$

The transverse shear force F and moments are shown in figure 3(c). From the geometry lengths l_1 and l_2 can be determined.

From the right angle triangle OMB

$$\sin\alpha = \frac{MB}{OB} = \frac{l_1}{R_m}$$

And

$$\cos\alpha = \frac{OM}{OB} = \frac{OM}{R_m}$$

Therefore,

$$l_1 = R_m \sin\alpha$$

And $l_2 = MN = ON - OM = R_m - R_m \cos\alpha$

i.e. $l_2 = R_m(1-\cos\alpha)$

Substuting for l_1 and l_2 in equation (1) and (2)

$$M_{XX} = FR_M \sin\alpha \tag{3}$$

$$T_{XX} = FR_m(1-\cos\alpha) \tag{4}$$

For any section X-X the extreme points R and S are critical. As at these points the transverse shear stress is zero and bending stress is maximum. And the expression for transverse

shear is given by $\tau = \frac{3}{2}\frac{F}{A}\left(1-\left(\frac{y}{r}\right)^2\right)$ (5)

τ = 0 for y= r, i.e. At point R and S.

Bending stress at R due to M_{XX} is

$$(\sigma_b)_{XX} = \frac{M_{XX}}{Z} = \frac{32FR_m \sin\alpha}{\pi d^3} \tag{6}$$

Shear stress due to twisting moment T_{XX} on the outer radius of the cross section

$$(\tau_t)_{XX} = \frac{T_{XX}}{Z_p} = \frac{16FR_m(1-\cos\alpha)}{\pi d^3} \tag{7}$$

Principal stresses at R are given by,

$$\sigma_{1,2} = \left[\frac{16FR_m}{\pi d^3}\right]\left[\sin\alpha \pm \sqrt{(\sin\alpha)^2 + (1-\cos\alpha)^2}\right]$$
$$\sigma_{1,2} = K\left[\sin\alpha \pm 2\sin\frac{\alpha}{2}\right] \tag{8}$$

Where, $K = \left[\frac{16FR_m}{\pi d^3} \right]$

LOCATIONS OF MAXIMUM STRESSES

Maximum Bending Stress

The condition for maximum bending stress is ,

$$\frac{\partial \sigma}{\partial \alpha} = 0$$

Substuting for 'σ' from equation (6) we get

$$\frac{32FR_m \cos\alpha}{\pi d^3} = 0$$

As F, R_m, d are non zero for a non trivial solution, above equation is satisfied when

$$\cos\alpha = 0$$

i.e.

$$\alpha = \frac{\pi}{2} radians \text{ or } \alpha = 90^0$$

Therefore the bending stress is maximum at the cross section making an angle $\alpha=90^0$ with respect to loading line.

The variation of bending stress with respect to α is shown in figure 4.

Maximum Shear Stress

The condition for maximum shear stress is ,

$$\frac{\partial \tau}{\partial \alpha} = 0$$

Substuting for 'τ' from equation (8) we get

$$\frac{16FR_m \sin\alpha}{\pi d^3} = 0$$

For a non trivial solution, above equation is satisfied when

$$\sin\alpha = 0$$

i.e.

$$\alpha = \pi radians \text{ or } \alpha = 180^0$$

Therefore the torsional shear stress is maximum at the cross section making an angle $\alpha=180^0$.

The variation of torsional shear stress with respect to α is shown in figure 4.

Maximum Principal Stress

The condition for maximum principal stress

$$\frac{\partial \sigma_1}{\partial \alpha} = 0$$

Substuting for 'σ_1' from equation (7) we get

$$0 = K(\cos\alpha + \cos\frac{\alpha}{2})$$

$$K = \left[\frac{16FR_m}{\pi d^3}\right] \neq 0$$

For a non trivial solution, above equation is satisfied when

$$(\cos\alpha + \cos\frac{\alpha}{2}) = 0$$

$$\cos\alpha = -\cos\frac{\alpha}{2}$$

$$\cos\alpha = \cos(180 - \frac{\alpha}{2})$$

$$\alpha = (180 - \frac{\alpha}{2})$$

$$\alpha = 120$$

From this we see that the principal stress is maximum at the section making angle of 120 degree. The variation of maximum principal stress with respect to α is shown in figure 4.

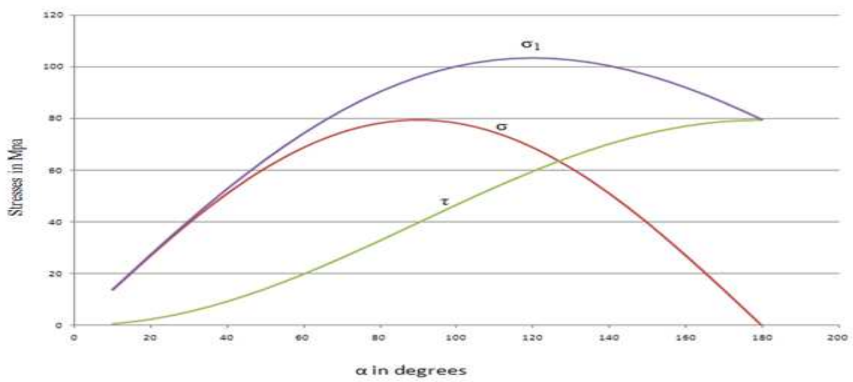

Figure 4: Plots According to Table 1

Table 1: Theoretical Values of Principle Stresses R_{mean} =125mm, d =20mm, F = 500 N

A(Deg)	Σb(Bending Stress) N/Mm²	Tt (Torsional Stress) N/Mm²	Σ1(Mpa)	Σ2 (Mpa)
10	13.82549	0.60448	13.84486	-0.02639
20	27.2309	2.399552	27.42701	-0.20993
30	39.80892	5.330675	40.4905	-0.7018
40	51.17736	9.308787	52.79276	-1.64139
50	60.9908	14.21302	64.11077	-3.15095
60	68.95107	19.89435	74.24672	-5.33068
70	74.81629	26.18016	83.03287	-8.25457
80	78.40826	32.87946	90.33559	-11.9671
90	79.61783	39.7887	96.05842	-16.481
100	78.40826	46.69794	100.144	-21.7756
110	74.81629	53.39724	102.5751	-27.7968
120	68.95107	59.68305	103.3741	-34.458
130	60.9908	65.36438	102.6015	-41.6417
140	51.17736	70.26861	100.354	-49.2026
150	39.80892	74.24672	96.76022	-56.9715
160	27.2309	77.17785	91.97698	-64.7599
170	13.82549	78.97292	86.18382	-72.3653
180	9.75E-15	79.5774	79.5774	-79.5774

RESULTS AND DISCUSSIONS

Results of above analysis are tabulated in Table 1 and plotted in figure 4. At any cross section making an angle ‘α’ the applied force ‘F’ induces transverse shear stress, torsional shear stress and bending stress. Magnitudes of these stresses will be varying over the cross section.

For extreme points R and S on the cross section of a diametral line perpendicular to plane of curved beam (the diametral line containing RS is also parallel to the loading line), the transverse shear stress is zero, bending and torsional shear stress are maximum. At the extreme points P, Q on the diametral line in the plane of curved beam the magnitude of resultant stress is much smaller and bending stress component is zero at these points. Therefore points R and S are the critical points and it is sufficient to determine stresses at R for different angles ‘α’ of the curved beam to obtain the variation of stresses.

Table 1 tabulates the values of bending stress (σ_b), torsional shear stress (τ_t), and maximum principal stress (σ_1) and minimum principal stress (σ_2) at points R for different values of α varying from 10^0 to 180^0. From the table it is clear that the magnitude of bending stress increases gradually from the loading point and becomes maximum for the section which makes an angle $\alpha = 90^0$ with loading end and then it decreases gradually and become zero at the fixed end. Torsional shear stress also increases gradually as α increase and is maximum at the fixed end. The values of principal stresses at the extreme points R on the cross section for different angles α shown that, the maximum principal stress σ_1 is tensile in nature and minimum principal stress σ_2 is compressive. Magnitude of maximum principal stress increases gradually from the loading and acquires maximum value at $\alpha = 120^0$ and then decreases and becomes equal to maximum torsional shear stress at the fixed end. The minimum principal stress acquires its minimum value at the fixed end. At the fixed end the magnitude of maximum principal stress is numerically equal to the minimum principal stress but is of opposite sign. This clearly indicates that at the fixed end of semi circular beam subjected to out-of-plane load a state of pure shear prevails.

The plots of above stresses at the critical point with respect to the angle α as shown in figure 4, substantiate the above results.

CONCLUSIONS

Stress analysis of a semi circular curved beam (α between 0 to 180 degree) fixed at one end and subjected to an out-of plane load is presented. It is shown that every section of the beam is subjected to a combination of transverse shear force, bending moment and a twisting moment. Maximum principal stress neither occurs at fixed end and nor at the farthest point from loading line. It occurs at a section $\alpha = 120^0$ with respect to free end. Also it is shown that a state of pure shear prevails at the fixed end. These results have important bearing on type of cross section that can be used for curved beams subjected to out-of-plane load.

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